on tensor rank entailed by the rapid growth of the number of tensor units as rank increases. Given that a short-term memory store of capacity 4 is connected to a tensor-like system for processing, the limit of 4 on store size is a consequence of the fact that for most cognitive tasks, processing of the objects in the store is a necessity.

The links between storage and processing phenomena are worth exploring. In section 2, Cowan argues that the unity of conscious awareness implies the contents of attended channels should be integrated or combined. Similarly, category clusters (discussed in sects. 2.7 and 3.4.2) imply a link between instances of the category. Cowan further contends, in section. 3.1.3, that the short term storage limit is observed only with items recalled in correct serial positions. Given that the slots of relation are identified, serial position can be coded as a relation ordered-items (item 1, item 2, item 3, item 4). The observation of no limit with free recall would then suggest that it is ability to represent the relation, rather than the items, that is subject to the limit. This would appear to be consistent with the relational complexity theory of Halford et al. (1998). Furthermore, it clearly points to explaining storage limits in terms of complexity of relations that can be represented. This would also explain the finding of Nairne (1991, referred to by Cowan in sect. 3.4.3) that errors occur up to three positions from the correct position. The reason would be that the items are represented as a quaternary relation, which contains only four slots. The further finding, in section 3.4.5 that participants could predict the seventh item from items 3, 4, 6 may also indicate that the task is represented as a quaternary relation.

These phenomena indicate links between entities that are important, but the nature of these links is not really clear, and the issue is clouded by the lack of a well specified theory in Cowan's paper. Some properties of relational knowledge defined by Halford et al. (1998) seem to be involved in the phenomena discussed above, but it is not clear that they all are. We could define the relational instances fruit (apple, banana, orange, pear) and fruit (lychee, pineapple, passionfruit, guava), and so on. Organizing memory storage as quaternary relations in this way would account for recall of items in clusters of four. However, it would also predict a lot of other properties of relational knowledge that Cowan has not demonstrated. For example, relational knowledge has the property of omni-directional access (Halford et al. 1998) which means that, given any $n-1$ components of a relational instance, the remaining component can be retrieved. Thus, given the quaternary relation proportion (4, 2, ?, 3) we can determine that the missing component must be "6" because it is necessary to complete the proportion 4/2 = 6/3. However, it is far from clear that category clusters share this property. If given a list [apple, banana, ? pear] there is no particular reason why we should recall "orange." Thus category clusters do not entail the kind of constraints that are entailed in relations. Another property of relational knowledge is that analogical mappings can be formed between corresponding relational instances (Holyoak & Thagard 1995). Again, it is not clear that analogies can be formed between category clusters.

Storage is not a simple, unitary matter, but can take many forms. Furthermore, the form in which information is stored affects the form in which it is processed. Some of the possibilities, together with possible implementation in neural nets, are:

1. Item storage – implemented as a vector of activation values over a set of neural units.
2. Associative links between items, implemented as connection weights between units in different vectors.
3. Superposition of items – implemented as summation of item vectors. This is tantamount to a prototype.
4. Superimposed items bound to a category label, such as fruit(apple) + fruit(banana) ± fruit(orange) + fruit(pear). This is equivalent to a unary relation and can be represented by a Rank 2 tensor:

$$v_{fruit} \times v_{apple} + v_{fruit} \times v_{orange} + v_{fruit} \times v_{orange} + v_{fruit} \times v_{pear}$$

Item-position bindings: ordered-fruit (first, apple) + ordered-fruit {(second, orange) +, . . . , + ordered-fruit (fourth, pear)}. This is a binary relational instance and can be implemented by the tensor product

$$v_{ordered\text{-}fruit} \times v_{first} \times v_{apple} + v_{ordered\text{-}fruit} \times v_{second} \times v_{orange} + \ldots + v_{ordered\text{-}fruit} \times v_{fourth} \times v_{pear}$$

Binding items into n-ary relations where n has a maximum value 4. This can be implemented by a tensor up to Rank 5:

$$v_{fruit} \times v_{apple} \times v_{orange} \times v_{pear}$$

These representations have different characteristics. They permit different retrieval operations, and impose different processing loads. More important, at least some of these properties can be captured by neural net models. The Rank n tensor would explain why processing load increases with the number of entities related, and consequently suggests why the capacity limit tends to be low. However, the earlier representations are not sensitive to processing load in this way. It should be clear from these examples that storage and process are intimately related, and that a theory of capacity must include both aspects of computation. However, while their interaction may be complex, it is not arbitrary. Our theory specifies a unique set of properties for processes involving relations of different antics.

Conclusion. Cowan has done the field a great service by showing that a broad range of observations is consistent with the limit of four entities that had been proposed previously by Halford et al. (BBS, 1998). However his claim to reduce processing capacity to storage capacity is not substantiated. Furthermore he offers no explanation for the limit, and glosses over the fact that at least one existing theory offers a potential explanation as to why the limit should be small.

Pure short-term memory capacity has implications for understanding individual differences in math skills

Steven A. Hecht and Todd K. Shackelford

Division of Science, Department of Psychology, Florida Atlantic University, Davie, FL 33314. **{shecht; tshackelford}@fau.edu**
www.uni-bielefeld.de/ZIF/heiko.html

Abstract: Future work is needed to establish that pure short-term memory is a coherent individual difference attribute that is separable from traditional compound short-term memory measures. Psychometric support for latent pure short-term memory capacity will provide an important starting point for future fine-grained analyses of the intrinsic factors that influence individual differences in math skills.

Cowan presents a clear and convincing theoretical case for the fixed capacity limit of three to five chunks in the focus of attention. Cowan has significantly advanced the field of memory research by providing a cogent analysis of the conditions that must be met in order for memory storage capacity to be measured accurately. Cowan presents an organized and impressive array of empirical data corroborating his theoretical claims. Cowan's claims rely on the idea that "purer" estimates of storage capacity in adults can be derived from existing sources of evidence. These sources of evidence come from various methodologies, and converge on the conclusion that a smaller than previously thought chunk limit exists in the focus of attention.

Cowan's analysis provides researchers with a promising tool for relating capacity limits to individual differences in various kinds of human abilities; in this commentary, we address mathematical thinking. Cowan distinguishes between memory measures that do, or do not, control for noncapacity-limited mechanisms. "Compound short-term memory" tasks capture both memory storage capacity and other sources of variance, such as strategic processing. An important limitation of traditional compound short-term memory tasks is that the relative contributions of both pure short-term memory and other noncapacity limited mechanisms to variability in math skills are usually not readily determined. Only compound memory tasks have been systematically investigated as capturing memory processes that contribute to individual differences in math skills (see Geary 1993, for a review)

There are at least three aspects of a complete account of individual differences in math skills (Hecht 1998). First, it is necessary to focus on specific subdomains of math skills (e.g., simple arithmetic, fraction estimation), because different factors may influence each subdomain of math ability. Second, the unique mathematical knowledge (e.g., counting knowledge) needed to carry out specific problems in a subdomain should be determined. Third, the contributions of intrinsic factors such as memory capacity on the efficiency with which mathematical knowledge is acquired and carried out should be investigated. Another characteristic of a complete account of variability in math skills is a description of the relative contributions of biological and environmental mechanisms that influence the development of math skills (Geary 1995). As Cowan alluded, pure short-term memory capacity may be determined solely by biological factors optimized by adaptive processes in human evolution. Quality of math instruction would be an example of an environmental mechanism.

Cowan's target article suggests important avenues of future research that might lead to a complete account of variability in math skills. The first suggested line of research is the nature of individual differences in pure short-term memory capacity. The relations between pure short-term memory capacity and variability in academic performance can be investigated only if pure estimates of capacity can be measured as a distinct individual difference attribute. Cowan notes that individual differences in capacity limits appear to exist in seemingly disparate tasks such as Sperling's (1960) full report task, Cowan et al. (1999) unattended speech task, and Luck and Vogel's (1997) visual storage capacity task. Performance on these seemingly disparate tasks should correlate if they measure the same underlying pure short-term memory construct.

Confirmatory factor analysis (CPA) can be used to support empirically pure short-term memory capacity as a unique domain of memory ability. Based on Cowan's analysis, it is likely that pure short-term memory and compound memory tasks will yield separate, though correlated, factors. The constructs should be correlated to the extent that compound short-term memory task performance is influenced by pure short-term memory capacity. An important benefit of using latent variables is that correlations among factors can be observed while controlling for sources of variance associated with task specific level and direction of effort or attention. CPA also can be used to determine whether separate versus central pure short-term memory capacity exists. For example, CPA can be used to determine if visual versus verbal pure capacity tasks yield separate or singular constructs of memory storage capacity.

If the construct validity in the psychometric sense of pure short-term memory tasks is empirically established, then another line of research could examine whether individual differences in the capacity limit of chunks in the focus of attention contributes to math ability. Although relations between memory-related latent factors and emerging variability in math outcomes have a been demonstrated (see Hecht et al., in press), these predictors are nor as fine-grained as the measures suggested by Cowan. That is, an important limitation of extant research relating memory processes to variability in math skills is that observed correlations do not indicate which aspects of memory performance influence specific aspects of math ability (Geary 1993). Once separate latent variables for pure short-term memory capacity and compound short-term memory have been identified, the relative contributions of these factors to individual differences in math skills can be assessed.

Thus, Cowan's analysis suggests important starting points for finegrained investigations of relations between intrinsic memory abilities and variability in specific aspects of mathematical ability. One place to start is suggested by Cowan's speculations regarding pure short-term capacity size and performance on Logan's (1988, experiment 4) alphabet arithmetic task. The alphabet arithmetic test is considered to be an analog measure of the acquisition of simple arithmetic knowledge (Logan 1988). Cowan speculated that problems with addends of 1–4 can be visualized (i.e., held in the focus of attention) more clearly while problem solving, because the addend sizes correspond to the number of chunks of information that can be held in the focus of attention. In contrast, performance on problems with addends of five or more may be hindered by pure short-term capacity limitations. Presumably, individual differences in pure short-term memory capacity should be associated with the effects of addend size on alphabet arithmetic performance.

Obtained relations between pure short-term memory capacity and variability in math skills also may help disentangle the influences of biological and cultural factors on math attainment. Geary (1995) makes a distinction between biologically primary abilities and biologically secondary abilities. Biologically primary abilities are found cross-culturally and are designed by natural selection in our evolutionary past. Biologically secondary abilities are not found in all cultures and require sustained formal training (e.g., reading, advanced calculus). It is likely that mean estimates of pure short-term memory capacity size, and degree of individual differences in that construct, are uniformly found across cultures. In his description of teleological accounts of a pure short-term memory capacity limit, Cowan reviews evidence from several sources suggesting a plausible evolutionary function of a very limited capacity of chunks in the focus of attention. For example, Cowan summarizes evidence by Kareev (1995) that a limited pure short-term memory capacity assists in the efficient detection of correlations between features in the physical world. A limited pure short-term memory capacity, shaped by evolutionary forces, may currently be "co-opted" for many contemporary tasks such as biologically secondary mathematical problem solving skills. Cowan's analysis suggests a line of research for investigating potential indicators of co-optation in the domain of math skills. Cooptation of biologically primary memory ability would be suggested by observed correlations between pure short-term memory capacity and biologically secondary math skills.

Cowan's target article should stimulate important avenues of future research toward demonstrating psychometric support for separate constructs of pure short-term memory and compound memory capacity. Current research focusing on individual differences in mathematical thinking has much to gain from the kind of fine-grained analyses of memory capacity suggested by Cowan. The predictive validity of latent pure short-term memory capacity would provide important progress toward understanding the biologically primary factors that influence variability in math skills.